HW 7

(1) Beats at 3 Hz (3 times per second).

(2) (a) $E = E_L + E_R = E_0\left(\sin(kx \mp \omega t) + \rho \sin(kx \pm \omega t)\right)$

→ Choose +x-axis going to right. Thus E_L moves in −x direction & E_R in +x :

$$E = E_0\left[\sin(kx+\omega t) + \rho \sin(kx - \omega t)\right]$$

$$= E_0\left[\sin(kx)\cos(\omega t) + \sin(\omega t)\cos(kx) + \rho\sin(kx)\cos(+\omega t) + \rho\sin(-\omega t)\cos(kx)\right]$$

$$= \sin(kx)\cos(\omega t)\cdot E_0\rho + \underline{E_0\rho\sin(kx)}\cos(\omega t)$$
$$\underline{- E_0\rho\sin(kx)\cos(\omega t)} + E_0\sin(kx)\cos(\omega t)$$
$$+ E_0\sin(\omega t)\cos(kx) - E_0\rho\sin(\omega t)\cos(kx)$$

$$= 2E_0\rho\sin(kx)\cos(\omega t) + E_0\sin(kx+\omega t)$$
$$- E_0\rho\sin(kx+\omega t)$$

$$E = 2E_0\rho\sin(kx)\cos(\omega t) + (1-\rho)E\sin(kx+\omega t)$$

(b) The first term $\left[2E_0 \rho \sin(kx) \cos(\omega t)\right]$ is a standing wave, while the second is a traveling wave: $\left[(1-\rho) E_0 \sin(kx+\omega t)\right]$

When $\rho = 1$, you get 100% reflection, and a ~~static~~ standing wave is formed.

When $\rho < 1$, the reflected light is less strong than the incoming light. You will see a smaller amplitude standing wave, but there will also be an "offset" from the incoming wave.

③ (a) $E_{arm_1} + E_{arm_2} = E_{01} e^{i(k\Delta x_1 + \omega t)} + E_{02} e^{i(k\Delta x_2 + \omega t)}$

$\rightarrow$ Assume $E_{01} \approx E_{02} \equiv E_0$

$\rightarrow$ $\Delta x_1 = 2L_1$ & $\Delta x_2 = 2L_2$

$$E_{total} = E_0 \left[e^{i(2L_1 k + \omega t)} + e^{(i 2L_2 k + \omega t)} \right]$$

$\rightarrow$ $k = \frac{2\pi}{\lambda}$

$$E_{total} = E_0 e^{i\omega t} \left[e^{i\left[\frac{4\pi L_1}{\lambda}\right]} + e^{i\left[\frac{4\pi L_2}{\lambda}\right]} \right]$$

(b) $$I \propto E^* E = E_0^2 e^{i\omega t} e^{-i\omega t} \left[e^{i[4\pi L_1/\lambda]} + e^{i[4\pi L_2/\lambda]} \right] \times \left[e^{-i[4\pi L_1/\lambda]} + e^{-i[4\pi L_2/\lambda]} \right]$$

$$I \propto E_0^2 (1) \left[e^0 + e^{-i\frac{4\pi}{\lambda}[-L_2 + L_1]} + e^{i\frac{4\pi}{\lambda}[L_1 - L_2]} + e^0 \right]$$

$$I \propto E_0^2 \left[2 + 2\cos\left(\frac{4\pi}{\lambda}(L_1 - L_2) \right) \right]$$

$\rightarrow$ $\Delta L = L_1 - L_2$

$\rightarrow$ $\propto$ sign allows us to divide by 2 (proportionality const.)

$$I \propto E_0^2 \left(1 + \cos\left(\frac{4\pi \Delta L}{\lambda} \right) \right)$$

(3 continued)

(c) $1 + \cos(x)$ has maxima or minima every $2\pi = \Delta x$:

$$\frac{4\pi \Delta L}{\lambda_1} - \frac{4\pi \Delta L}{\lambda_2} = 2\pi$$

$$\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{2\pi}{4\pi \Delta L} = \frac{1}{2\Delta L}$$

$$\frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2} = \frac{1}{2\Delta L}$$

$\rightarrow$ Let $\Delta\lambda = \lambda_2 - \lambda_1$ and $\lambda_1 \approx \lambda_2 \equiv \lambda$:

$$\Delta\lambda = \frac{\lambda^2}{2\Delta L}$$

(d) Resolution is $\Delta\lambda$ (part c)

$$\Delta\lambda = \frac{(780.00\,nm)^2}{2(1.00 - 0.10)m} = 3.38 \times 10^{-13}\ m = 0.338\ pm$$

$$\Delta\nu = \nu_1 - \nu_2 = \frac{c}{\lambda_1} - \frac{c}{\lambda_2} = \frac{c}{2\Delta L}$$

$$\Delta\nu = 167\ MHz$$

(3 continued)

(e) Resolution $\Delta\nu = \dfrac{c}{2\Delta L}$

- If you want better (i.e. smaller) resolution, ΔL should be made bigger (i.e. the interferometer arms are more unequal in length).

- ~~[illegible]~~ On the other hand, you can leave the arms so unequal that the light is no longer able to interfere with itself. This is given by the coherence time & length:

$$\Delta L_c = c\Delta T_c \quad \text{(coherence time \& length)}$$

Total difference in OPLs $< \Delta L_c$

$$\boxed{2(L_1 - L_2) < \Delta L_c}$$

This will limit the possible resolution you can achieve.

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(a) The P.D. will measure the beat between $f_{shifted}$ and f_0 :

$$f_{shifted} - f_0 = f_0\left[\frac{V + V_m}{V - V_m} - 1\right]$$

← use $v = c$ (vacuum)

$$= f_0 \left[\frac{c + v_m - c + v_m}{c - v_m}\right]$$

$$\boxed{\text{Beat} = \Delta f = f_0 \left(\frac{2 v_m}{c - v_m}\right)}$$

(b) $\lambda = 550 \text{ nm}$

$$f_0 = \frac{c}{\lambda} = 545 \text{ THz}$$

$$v_m = 10 \text{ cm/s}$$

$$\text{Beat} = (545 \times 10^{12} \text{ s}^{-1}) \frac{2 \cdot 10 \text{ cm/s}}{3 \times 10^8 \text{ m/s} - 0.10 \text{ m/s}}$$

$$= \boxed{364 \text{ kHz}}$$

(5) $$E = E_0 (1 + a \cos w_m t) \cos w_c t$$

$$= E_0 \cos w_c t + E_0 a \cos w_m t \cos w_c t$$

$$= E_0 \cos w_c t + E_0 a \left[\frac{\cos[(w_m + w_c)t] + \cos[(w_m - w_c)t]}{2} \right]$$

$$E = E_0 \cos(w_c t) + \frac{E_0 a}{2} \left(\cos[(w_c + w_m)t]\right) + \frac{E_0 a}{2} \cos[(w_c - w_m)t]$$

You want side bands from 20 Hz to 20,000 Hz (20 kHz), you need this range on both sides of carrier.

$$\boxed{\Delta w \geq 40 \text{ kHz}}$$

E.C.

(a) $v_g = \frac{d\omega}{dk} = \frac{d}{d(2\pi/\lambda)}(2\pi\nu) = \frac{d\nu}{d(1/\lambda)}$

$\rightarrow \nu = \frac{c/n}{\lambda}$

$\frac{d\nu}{d(1/\lambda)} = \frac{c}{n} + \frac{c}{\lambda} \cdot \frac{d}{d(1/\lambda)}\left[\frac{1}{n}\right]$ ← Chain Rule

(b) phase velocity $= v_{ph} = \frac{c}{n}$

$v_g = v_{ph} + \frac{c}{\lambda}\frac{d[1/n]}{d[1/\lambda]}$

If $v_g < v_{ph}$, then $\frac{d[1/n]}{d[1/\lambda]}$ must be negative.

- Let $y = \frac{1}{\lambda}$. From plot, we see $n(\lambda) \approx \frac{|A|}{\lambda}$ (a good guess at the functional form). Thus, $\frac{1}{n} \approx \frac{\lambda}{|A|}$.

$\frac{d}{d[1/\lambda]}\left[\frac{1}{n(\lambda)}\right] \approx \frac{d}{dy}\left(\frac{1}{A} \cdot \frac{1}{y}\right) = \frac{-1}{|A|y^2} = \frac{-1}{|A|} \cdot \lambda^2$

This term is negative! $\Rightarrow v_g < v_{ph}$